Modeling feeds of antennas by Finite Element Method

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The simulation of the input impedance and the radiation pattern of an antenna is very important in the design stage of antennas when engineers use computer aided design software. Other parameters can be calculated from these data.
The simulated input impedance depends on the applied feed model that is the reason why it is so important to know the behaviors of the different feeding models. The most frequently used models are prescribed in the paper in the frame of the Finite Element Method. The models can be used accurately in other numerical techniques as well.

1. Introduction

The most important measured parameters of an antenna are the input impedance and the radiation pattern. Other parameters, for example the reflection coefficient or the voltage standing-wave ratio can be calculated from the input impedance, the directivity as well as the gain can be obtained from the radiation pattern. The simulated input impedance depends on the applied feed model that is the reason why it is so important to know the advantages and the disadvantages of the feeding models. The most frequently used models are the current probe model, the voltage gap generator, the magnetic frill generator and the waveguide port.

This paper presents the above mentioned approaches through a monopole antenna situated above a ground plane. The Finite Element Method (FEM) has been used in the numerical field analysis, which is a widely used technique to solve partial differential equations obtained from Maxwell's equations. Here, the Helmholtz-equation for the magnetic field intensity is studied in two dimensions supposing axial symmetry. First, the problem and the corresponding equations are shown, and then the four feeding models are described. After the presentation of numerical results, a short discussion and summary close the paper.

2. Finite Element Method (FEM) in antenna simulation

The FEM is a widely used numerical technique in computer aided design of electrical engineering problems. Only a brief introduction can be provided here, a detailed description can be found in [1-3].

The basis of the technique is the discretization of the problem region by simple elements. These finite elements are the triangle and the quadrangle in two dimensional problems, or the tetrahedral, hexahedral and prism elements in three dimensional problems. The system of equations to be solved for the potentials or for the field quantities can be assembled after obtaining the weak formulation of the partial differential equations and the boundary conditions of the problem. The latter equations can be derived from Maxwell's equations [1-4].

The problem to be presented here is a monopole antenna situated above a ground plane [1]. The body of the antenna is the inner wire of a coaxial transmission line as it can be seen in *Fig. 1*. The following Maxwell's equations must be solved in the domain Ω [4]:

$$\nabla \times \boldsymbol{H} = j\omega\varepsilon\boldsymbol{E}, \quad (1)$$

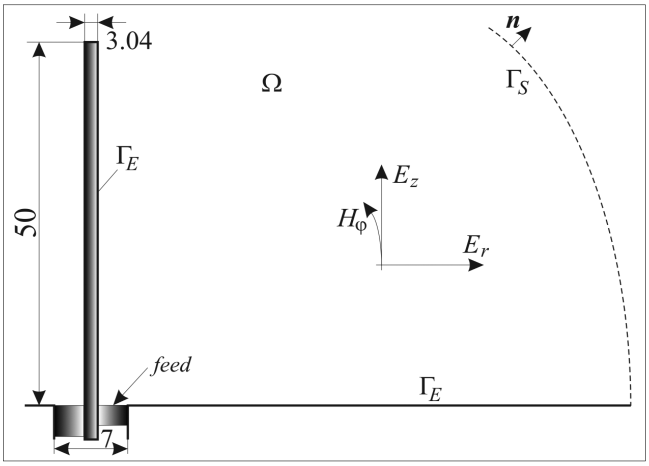
$$\nabla \times \boldsymbol{E} = -j\omega\mu\boldsymbol{H}, \quad (2)$$

$$\nabla \cdot \boldsymbol{H} = 0, \quad (3)$$

$$\nabla \cdot \boldsymbol{E} = 0, \quad (4)$$

where **H**, **E**, ω, ε, and μ are the magnetic field intensity and the electric field intensity, the angular frequency of excitation, the permittivity and the permeability, respectively. The phasor representation has been used, because of the time-harmonic representation (the excitation is supposed to be sinusoidal), i.e. $j=\sqrt{-1}$ is the imaginary unit.

Figure 1. The geometry of the monopole antenna



It is well known that the electromagnetic field of the monopole antenna is transverse magnetic (TM) [1,2,4], i.e. the magnetic field has only one component in the φ-direction, and the electric field has two orthogonal components, as it is denoted in Fig. 1.

The electric field intensity must be normal to the surface of the ground plane and the surface of the antenna, i.e. the boundary condition

$$\boldsymbol{E} \times \boldsymbol{n} = \boldsymbol{0} \tag{5}$$

can be supposed on Γ_E. Here n denotes the outer normal unit vector.

On Γ_S, absorbing boundary condition must be prescribed to absorb the electromagnetic energy [1,2],

$$\lim_{r\to\infty} r\left[\nabla \times \boldsymbol{H} + \mathrm{j}k_0\, \boldsymbol{n} \times \boldsymbol{H}\right] = \boldsymbol{0}, \text{ on } \Gamma_S, \tag{6}$$

which can be approximated by the first order absorbing boundary condition

$$\boldsymbol{n} \times \left[\nabla \times \boldsymbol{H} + \mathrm{j}k_0\, \boldsymbol{n} \times \boldsymbol{H}\right] = \boldsymbol{0}, \text{ on } \Gamma_S, \tag{7}$$

where $k_0^2 = \omega^2\mu\varepsilon$ is the wave number in free space ($\mu = \mu_0$, $\varepsilon = \varepsilon_0$). This models the unbounded space. The calculation domain must be truncated somehow, because the discretization cannot be performed at infinity, so the condition (7) on Γ_S is available to decrease the domain volume. The efficiency of absorbing the electromagnetic energy along the boundary Γ_S can be increased by applying a perfectly matched layer (PML) which outer boundary has been assigned as the absorbing boundary [1].

Finally, $H \times n = 0$ must be satisfied along symmetry planes (along the line $r = 0$ in axial symmetry situations).

It is evident that the application of the magnetic field intensity as the primary variable results in the most economic formulation. The partial differential equation to be solved for the magnetic field intensity here is the following [1,4]:

$$\nabla \times \nabla \times \boldsymbol{H} - k_0^2 \boldsymbol{H} = \boldsymbol{0}, \tag{8}$$

and $E = (\nabla \times H)/\mathrm{j}\omega\varepsilon$ is the electric field intensity from (1) and (2). After some mathematical manipulations and using (3), the following partial differential equation can be obtained for H_φ:

$$\Delta H_\varphi + k_0^2 H_\varphi = 0, \tag{9}$$

which is a scalar Helmholtz-equation of the magnetic field intensity.

3. Feeding Models in FEM

The feeding models of antennas are applied to take the input of the antenna into account. The most widely used feeding models are shown in *Fig. 2* [1,5-7].

3.1 The current probe model

The most widely used current probe model is a short current with a delta function, e.g.

$$\boldsymbol{J}(x,y,z) = \boldsymbol{e}_z\, I_0\, \delta(x - x_f, y - y_f),\ \ 0 \le z \le d. \tag{10}$$

It models a wire with zero diameter, x_f and y_f are the coordinates of the current I_0 ($x_f = 0$ and $y_f = 0$ in Fig. 2/a), and J has only one component in the z direction. This infinitesimal dipole can be generalized in any direction of the space. The electromagnetic field is singular in the vicinity of the probe [1]. This is the reason why it is more convenient to prescribe the magnetic field intensity on the surface of the antenna wire as it is represented in Fig. 2/a. The φ-component of the magnetic field intensity can be calculated by

$$H_\varphi = \frac{I_0}{2a\pi},\ \ 0 \le z \le d, \tag{11}$$

and $a = 1.52$ mm is the radius of the antenna. The length of the probe in the z direction should be as small as possible, but it can be concluded that $d << \lambda$ must be specified, and λ is the wavelength of the electromagnetic wave in vacuum, $\lambda = c/f$ (c is the speed of light and f is the frequency of excitation).

Once the electric field E is determined by the applied numerical method, the voltage across the probe can be computed as

$$U = -\int_0^d E_z(\mathrm{r} = \mathrm{a})\,\mathrm{d}z, \tag{12}$$

and the input impedance of the antenna is

$$Z = U/I_0. \tag{13}$$

The current distribution along the antenna can be calculated by the following form of Ampere's law:

$$I(z) = 2a\pi\, H_\varphi(z). \tag{14}$$

3.2 The voltage gap generator

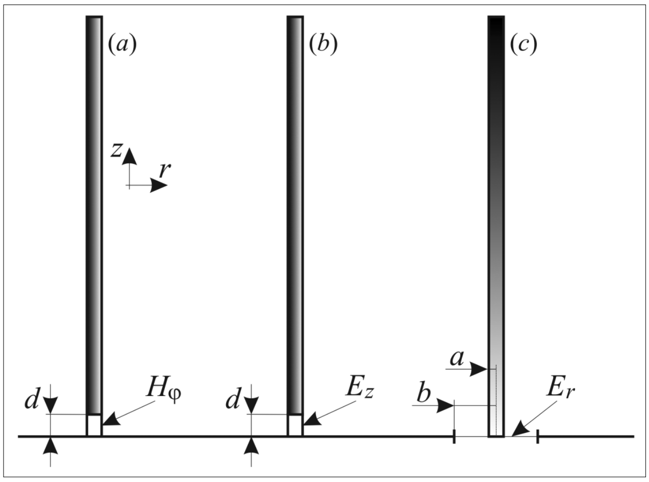
This model is basically used in the Method of Moments (MoM) [5], and it is based on the potential difference U_0 between the antenna and the ground plane as it is presented in Fig. 2/b. The z component of the electric field can be obtained by

$$E_z = -\frac{U_0}{l} \tag{15}$$

in the gap ($0 \le z \le d$), and d the length of the gap. The electric field intensity is prescribed along the line $r = a$, and $0 \le z \le d$ (see in Fig. 2/b).

The current in the feeding point then can be calculated by (14) substituting $z = 0$, i.e. $I = 2a\pi\, H_\varphi(z = 0)$, then the input impedance can be obtained by $Z = U_0/I$. The current distribution along the antenna can be simulated by (14).

Figure 2. Feeding models



3.3 The magnetic frill generator

The magnetic frill generator is a model of the antenna input fed by a coaxial line [5]. The following electric field intensity can be supposed in the radial direction by assuming purely TEM mode inside the coaxial transmission line (see in Fig. 2/c):

$$E_r(r) = \frac{1[\mathrm{V}]}{2r\ln(b/a)}, \quad a \le r \le b, \tag{16}$$

where a and b are the inner and outer radius of the coaxial line (a = 1.52 mm and b = 3.5 mm). The current distribution along the antenna can also be simulated by (14), and the input impedance can be calculated in the same way as presented in Section 3.2.

3.4 The waveguide port

The waveguide port model is more accurate and is a more efficient approach in general case. This is based on the weighted sum of TEM, TE and TM waveguide modes, and the weighting coefficients are collected in tables [1]. This model has been implemented in Comsol Multiphysics [8]. The scattering parameter (reflection coefficient) S_{11} can be extracted from the simulated electric field, finally, the input impedance can be obtained as [1,8]

$$Z = Z_0 \frac{1 + S_{11}}{1 - S_{11}}, \tag{17}$$

where $Z_0 = 50\,\Omega$ is the characteristic impedance of the waveguide. The reflection coefficient is calculated automatically in Comsol when applying waveguide port.

4. Simulation results

The problem has been solved by the functions of the Radio Frequency module of Comsol Multiphysics [8]. This software is a very efficient FEM design environment. The aforementioned feeding models can be implemented and tried out in an easy way. The models can be downloaded from the author's homepage [9].

The φ-component of the magnetic field intensity has been simulated by the TM Electromagnetic Waves application mode, and two dimensional axial symmetry plane has been analyzed for simplicity, because the aim is the study of the different models. Second order Lagrange shape functions have been used to approximate the unknown field quantity.

After some trials, 55296 triangles have been used to mesh the geometry *(Fig. 3)*, and it results in 111329 unknowns. This is a very dense mesh. The convergence of the simulated input impedance can be seen in *Fig. 4*, where the measured impedance is also shown. Measured data are from the paper [6]. The variation of input impedance is practically the same when applying finer and finer mesh. There is a permanent difference between measured and simulated data.

The geometry of the antenna has been subtracted from the calculation domain, because it is supposed to be made of ideal conducting material, i.e. discretization is not necessary inside the wire. The same mesh has

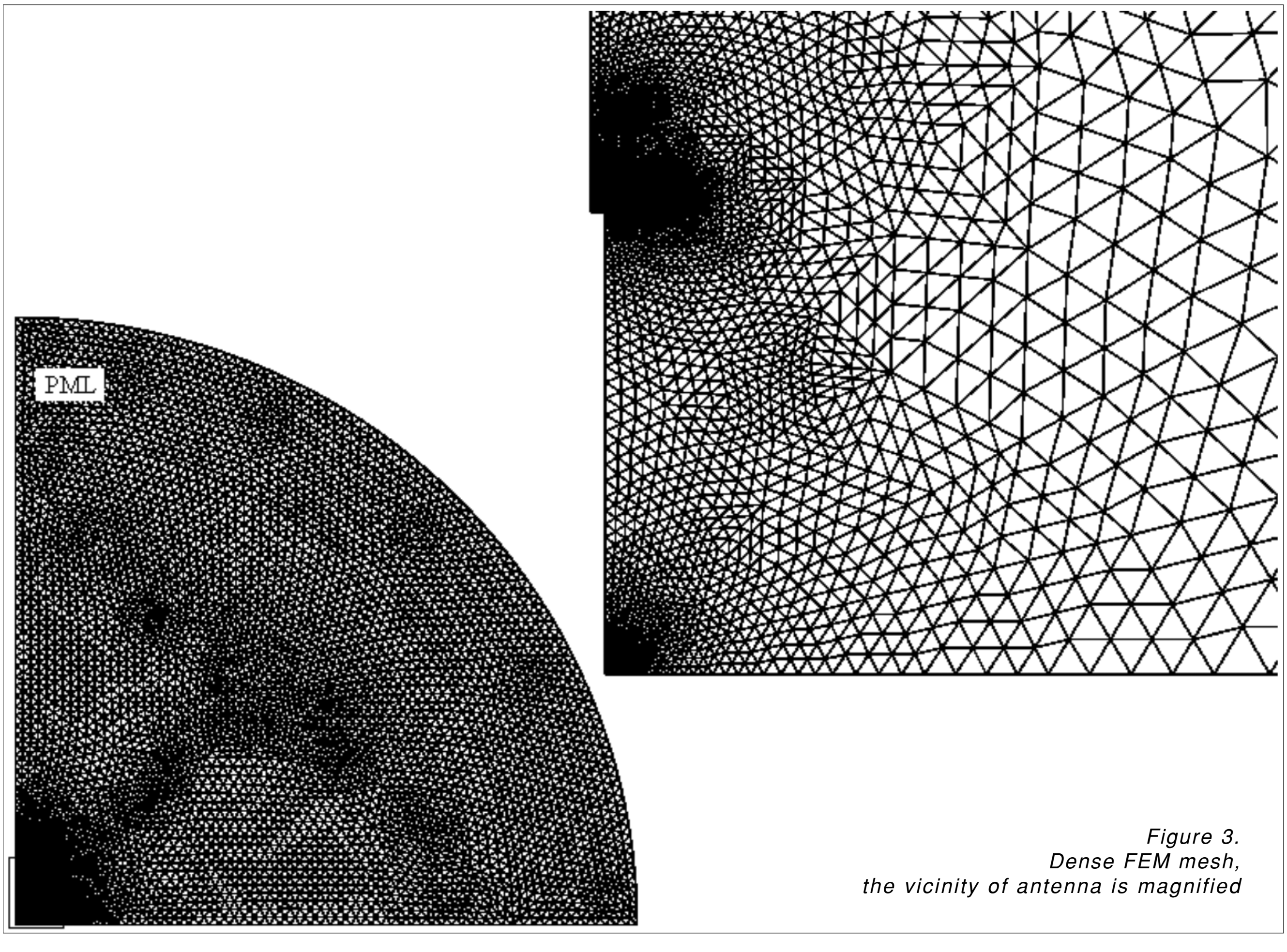


Figure 3. Dense FEM mesh, the vicinity of antenna is magnified

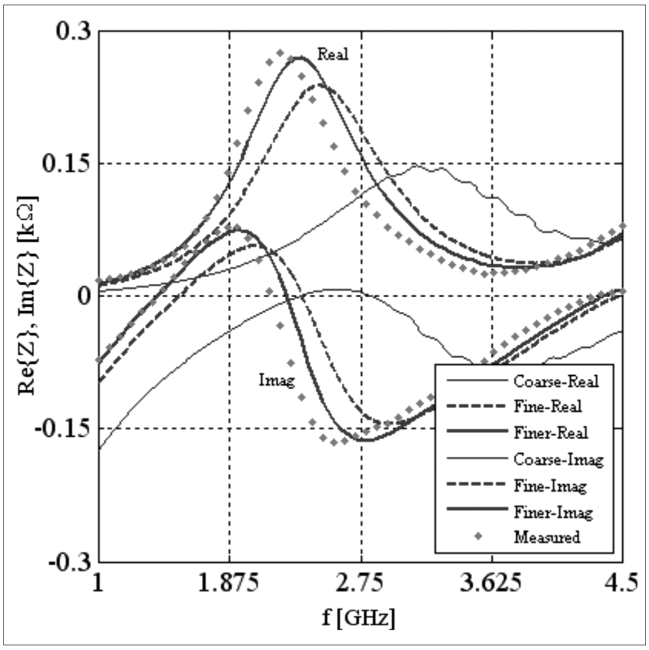


Figure 4.
Convergence of the solution vs. number of triangles

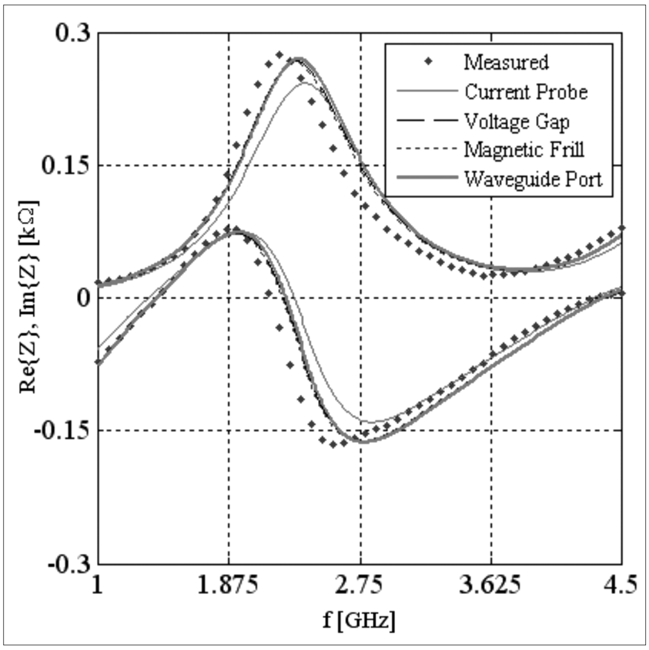


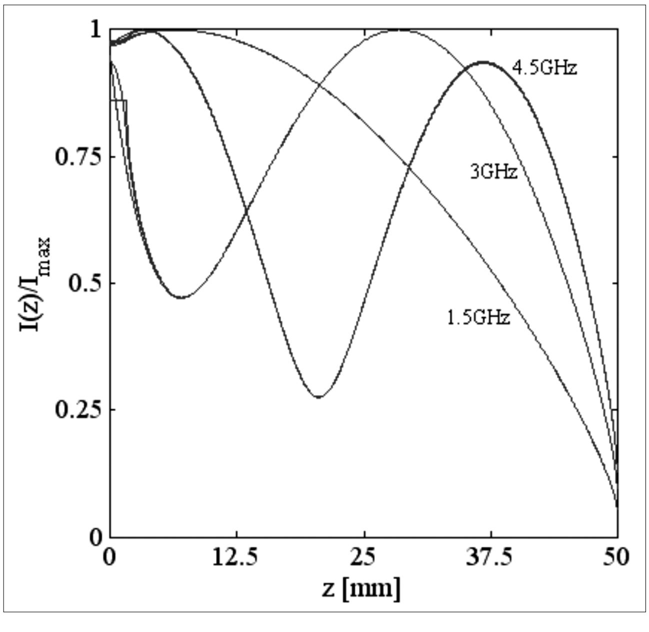
Figure 5.
Comparison of the input impedance of the monopole antenna

been used in all the frequency during the frequency sweep in the range of 1 GHz and 4.5 GHz. A PML layer has been inserted to improve the absorption of electromagnetic field, and the radius of the computational domain is 1 m.

Fig. 5 shows a comparison between measured input impedance and simulated ones. The application of current probe model results in the weakest approximation, the approximated value obtained from the other models are practically the same.

The current distribution along the antenna is a very important input data to calculate other important quantities.

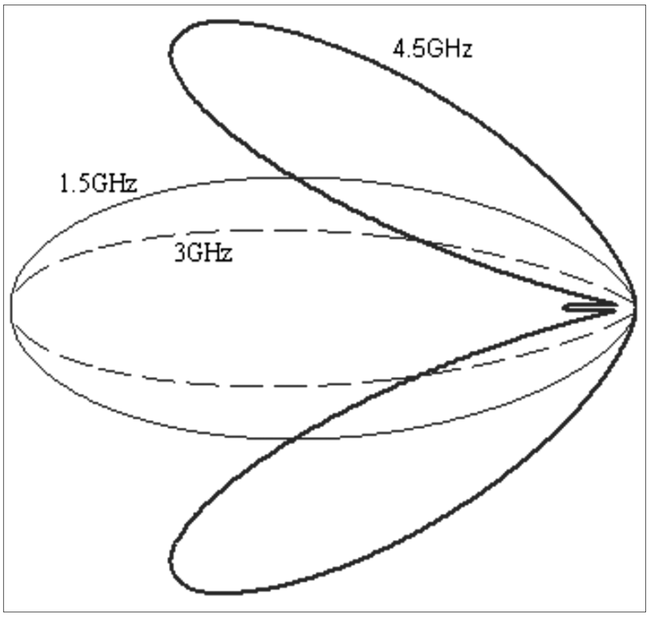
Figure 6.
Normalized current distribution along the antenna at three different frequencies



A comparison between the obtained currents simulated by the above mentioned feeding models can be seen in *Fig. 6* at the frequencies f = 1.5 GHz, 3 GHz and 4.5 GHz. The results are practically the same, but a small difference can be seen in the vicinity of z = 0 (the feeding point), and it is the effect of the different feeding models.

Fig. 7 shows the simulated field pattern of the monopole antenna at three different frequencies. The characteristics have been mirrored to the plane. It is noted that the characteristics simulated by the different feeding models are practically the same, i.e. the far field region is not depending on the applied feeding model.

Figure 7.
The normalized far field characteristics at three different frequencies



5. Summary

Feeding models of antennas have been presented in the frame of FEM. The input impedance, the current distribution and the characteristics of a monopole antenna on a ground plane have been simulated and compared with measured data. The next step of the research work is to apply the feeding models in the case of more complex antennas in 3-dimensional situations, and to compare the results with other numerical techniques, e.g. with MoM.

Author

MIKLÓS KUCZMANN was born in Hungary, 1977. He has become M.Sc. in Electrical Engineering in 2000, and Ph.D. in Electrical Engineering in 2005 at the Budapest University of Technology and Economics, Department of Electromagnetic Theory. He is Associate Professor at the Department of Telecommunications, Széchenyi István University, Győr, Hungary, where he is the Head of Laboratory of Electromagnetic Fields since 2005. Dr. Kuczmann has won the Bolyai János Scholarship from the Hungarian Academy of Sciences in 2006, the “Best PhD Dissertation” award in 2006 from the same Institute, and the Award of Academic Press in 2010.

References

[1] J.-M. Jin, D. J. Riley,
Finite Element Analysis of Antennas and Arrays,
Wiley, IEEE Press, 2009.
[2] J.-M. Jin,
The Finite Element Method in Electromagnetics,
Wiley, IEEE Press, 2002.
[3] M. Kuczmann, A. Iványi,
The Finite Element Method in Magnetics,
Academic Press, Budapest, 2008.
[4] K. Simonyi, L. Zombory,
Theoretical electromagnetics,
Műszaki Könyvkiadó, Budapest, 2000.
[5] W.C. Gibson,
The Method of Moments in Electromagnetics,
Chapman & Hall/RCR, 2008.
[6] T. Hertel, G. Smith,
On the convergence of
common FDTD feed models for antennas,
IEEE Trans. Antennas Propag.,
Vol. 51, No. 8, pp.1771–1779, 2003.
[7] Z. Lou, J.-M. Jin,
Modeling and Simulation of Broad-Band Antennas
Using the Time-Domain Finite Element Method,
IEEE Trans. On Antennas Propag.,
Vol. 53, No. 12, pp.4099–4110, 2005.
[8] http://www.comsol.com
[9] http://maxwell.sze.hu/~kuczmann/
Korszeru_antenna/Antenna_lap.htm