Mathematical algorithms of an indoor ultrasonic localisation system

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Our indoor ultrasonic localization and motion-tracking system (BATSY) is based on a mobile node capable of emitting radio and ultrasonic signals and a number of ultrasound sensors mounted at known positions in a room. The system uses the node's distance to the sensors (as derived from the arrival times of the ultrasound signal) for calculating its position.
In this paper we discuss the mathematical and measurement problems related to ultrasonic localization, we propose a possible solution algorithm, and we present a method for determining the sensors' position in an automated way.

1. Introduction

As part of the AAL (Ambient Assisted Living) programme, we have developed an indoor ultrasonic localisation system at BAY-IKTI (Institute for Applied Telecommunication Technologies of the Bay Zoltán Foundation).

Such a system can be used in many fields, e.g. for monitoring the daily routine of injured or elderly people [1], or for tracking the movement of customers in a supermarket in order to observe and analyze their shopping habits. A similar approach has been applied to the problem in a number of research projects worldwide, see "The Bat Ultrasonic Location System" developed at Cambridge University [2], or the "Ultrasonic Localisation System" developed at HomeLab, Lucerne University [3].

A common shortcoming of such localisation systems based on ultrasonic distance measurement is, however, that in order to achieve adequate accuracy (in the 3 to 15 centimetres range), one needs to map the exact position of the sensors with much higher precision (in the sub centimetre range). The process for this kind of high-precision positioning, which must be performed prior to the first use of the system, can be technically demanding (it is usually done either using a conventional tape measure or with a laser range finder), hence adding substantially to the installation time of the system.

In contrast with the above difficulties, the method we have developed allows the sensors' positions to be determined quasi-automatically (i.e. without any preliminary positioning, either manual or instrumental) through the accurate measurement capacity of our localisation system.

The rest of the paper is organized as follows. In Section 2 we introduce our BATSY system, in the next section we discuss how to determine the position of a point in space with its distance given from a number of known points, in Section 3 we review and correct some of the positioning errors resulting from possible distortions in the ultrasonic distance measurement process, and finally, in Section 4 we propose a method to obtain the sensors' positions in an automated way.

2. Description of the BATSY system

We have started to build our ultrasonic localisation system in 2006 to develop a localisation and motion tracking tool for our AAL (Ambient Assisted Living) laboratory. Since the system relies on distance measurement based on the speed of an ultrasound signal, it has been named BATSY (BAT SYstem). As a result of various improvements made to the calculation method, we have managed to reach an accuracy of 3 centimetres.

Figure 1. The Batsy mobile node

The BATSY ultrasonic localisation system consists of three main components:

1. A number (6-8) of sensor units mounted on walls, for receiving ultrasound signals.
2. A computer equipped with a radio module, for receiving radio signals and performing calculations.
3. A mobile node, capable of emitting radio and ultrasound signals simultaneously *(Figure 1)*.

The underlying concept of the system is based on the difference between the speed of sound and that of a radio signal. A radio signal emitted by the mobile node arrives to the computer almost instantly, while the 40 kHz ultrasound wave dispatched at the same time travels at the speed of sound, thus it reaches the sensors mounted on the walls significantly later. Therefore, through measuring the difference between the arrival times of the radio and the ultrasound signals to some given sensor, one can calculate the distance between the mobile node and the sensor. Assuming we know the exact location of the sensors as well as the distances between them and the mobile node, we can calculate the position of the mobile node.

3. Trilateration: calculating the position of a point in space with its distance given from three other points

Assume we have three known points in space with coordinates $S_1=(x_1,y_1,z_1)$, $S_2=(x_2,y_2,z_2)$, $S_3=(x_3,y_3,z_3)$ respectively, and we further know their distances, d_1,d_2,d_3 from some unknown point (x,y,z). This unknown point is located at the intersection of three spheres with S_1,S_2, S_3 as their centres and d_1,d_2,d_3 as their radii, respectively. This method is based on the measurement of three distances, so we call it *trilateration*. Three spheres intersect in two points generally, and these two points of intersection are symmetrical with respect to the plane through S_1,S_2,S_3. The two intersection points are obtained as the solution to the following system of equations:

$$(x_1-x)^2+(y_1-y)^2+(z_1-z)^2=d_1^2$$
$$(x_2-x)^2+(y_2-y)^2+(z_2-z)^2=d_2^2$$
$$(x_3-x)^2+(y_3-y)^2+(z_3-z)^2=d_3^2$$

In the course of the actual calculations, differences like $(x_1-x_2),(z_2-z_3),...$, appear in the denominator, which causes a problem in case one of them is zero. In practice this happens quite often, since the sensors are usually mounted on walls and ceilings, thus some of their coordinates x, y, z are likely to coincide. The easiest way to avoid this problem is through adding small, independently chosen random numbers (in the range of a thousandth millimetre) to the coordinates. This practically does not decrease the accuracy of the calculations, while it avoids division by zero with a probability high enough.

In particular adaptations there is often an obvious opportunity for selecting the correct solution (i.e. the one corresponding to the actual location of the mobile node) out of the two candidates. For example, in case the three sensors are located on the ceiling of a room, the two solutions will fall on opposite sides of the ceiling, so it is straightforward to choose the one inside the room. When there is no chance for a solution of this kind, one can use the algorithm discussed in Section 4.1.

4. Correcting positioning errors resulting from distortions in distance measurement

In practical use, one has to apply different modifications to the theoretical algorithm described in Section 3.

4.1 Dealing with distortions in distance measurement

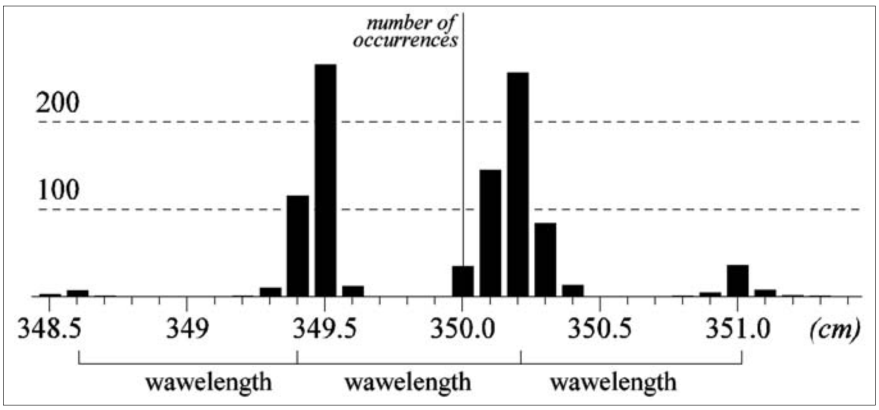
Sometimes, depending on the position of the mobile node, an obstacle along the straight line connecting the mobile node to a sensor may get in the way of the ultrasound wave. In that case, sound cannot reach the sensor along a direct path, hence distance measurement fails. This would not be of any problem in case no result was obtained from such occurrences of measurement; but what usually happens is that the sound wave reaches the sensor along a longer, indirect path. Thus the sensor detects a reflected signal, resulting in an estimated distance greater than the real one.

In order to avoid a possible localisation error caused by corrupted distance values, one should use a greater number (6-8) of sensors. This way, the estimated location of the mobile node would be obtained as the intersection of more than three spheres. However, there could still be errors made in the localisation of the centres and the measurement of the radii of the spheres (even additional to the previously mentioned ones), in which case more than three spheres may have no intersection at all.

Both problems can be dealt with simultaneously using an “Adaptive Fuzzy Clustering” algorithm [4].

The algorithm consists of two phases. The first phase involves calculating the intersection of all sphere-triplets derived from the measured distances, using the trilateration algorithm described in Section 3. If we have measured n distances, this will give $\binom{n}{3}$ possible locations as a set of points in space. Assuming the number of more or less accurate measurements is sufficiently high, the set of intersection points belonging to spheres with a correctly measured radius should be concentrated within a relatively small space segment, whereas the intersection points derived from one or more erroneously determined spheres will be dispersed in space essentially randomly.

Figure 2. Occurrences of different measured distances from 1000 measures at a distance of 350 cm



The second phase consists of finding the point with the maximum density within the above derived set of possible locations. This is done by calculating the vector average of the points, or the “centre of gravity” of the set. Next, we calculate the average distance of all points from this centre of gravity, and eliminate those whose distance is greater than average. In this way, we will have obtained a smaller set with which to repeat the second phase. We do the repetition until the diameter of the set falls below a required threshold value.

4.2 Theoretical and practical accuracy of distance measurement

The BATSY system uses 40 kHz ultrasound; its wavelength is about 8.6 mm in room temperature. The sensor we used detects the pressure peaks of air waves, so the arrival of an ultrasound packet will presumably be recorded at a pressure peak. These peaks lie 8.6 millimetres apart from each other, which adds an uncertainty factor of 8.6 mm to distance measurement.

The sensor’s sampling frequency – in accordance with the clock pulse of its microcontroller, and taking as given the speed of sound – translates into a sub-millimetric measurement accuracy.

In *Figure 2* we can see the results from 1000 individual measurements performed at a given distance. One can clearly see the occurrence peaks situated 8.6 millimetres away from each other.

4.3 Distortions related to the directional angles

The sensors and emitters are directed in space. This means that the distance between the sensor and the mobile node is measured correctly as long as one is positioned facing the other, but when one or both of them are rotated, the measured distance increases with the angle of their axes. This results in a systematic bias related to the sensor’s and the mobile node’s directional angles *(Figure 3)*.

Figure 3. The sensor’s (α) and node’s (β) directional angles

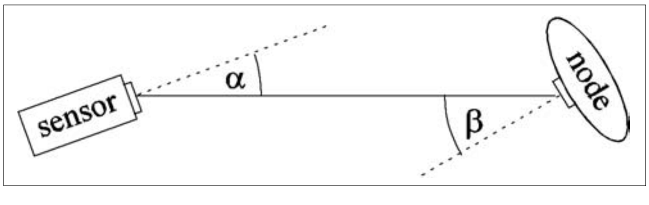
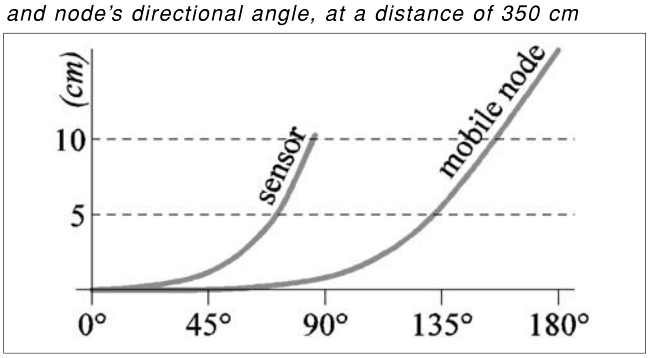


Figure 4.
Bias in distance measurement as a function of the sensor’s and node’s directional angle, at a distance of 350 cm



To offset the bias related to the sensor’s directional angle, we have to measure it as a function of the angle and distance variables *(Figure 4)*.

In order to eliminate the distortion from the measured distance, one needs to know the approximate distance of the mobile node as well as the angle between the sensor-node line and the sensor’s axis.

If the direction of the sensor’s axis is known beforehand (i.e. it was recorded at the moment of the sensor being mounted), the bias related to the sensor’s directional angle can be offset using the following two-step algorithm.

In the first step we determine the location of the mobile node using the method discussed in Section 3. This location will be inaccurate as yet, since it will contain a directional bias, but the deviation from the real position is not significant. So this inaccurate estimated location is suitable for calculating the angle between the sensor-node line and the sensor’s axis.

In the second step we calculate the range correction value for the given (angle, distance) pair and subtract it from the previously measured distance. Do it for all the sensors, and recalculate the location of the mobile node using the method specified in Section 3. This new location will be free of any directional angle effect.

We have tested the method in our laboratory and found that the distance between the positions calculated with and without the sensor’s directional angle correction is usually less than 20 millimetres. So if such accuracy is not required or the system is low on CPU performance, it can be omitted.

Correcting the bias related to the mobile node’s directional angle is only feasible if the direction of the node’s ultrasound emitter can be determined. In this case, one can use the same method as the one discussed above. In our specific application however, the mobile node’s direction was not fixed, so we could only measure the localisation error related to the sensor’s directional angle. Our chosen approach then was to place the mobile node to a pre-specified location and rotate it around while simultaneously calculating its indirectly estimated position through the above described algorithm. We have found the difference between the real and the calculated positions of the node to fall in the 0 to 45 mm range.

4.4 Distortions resulting from variations in the speed of sound

The speed of sound varies with temperature, humidity and air pressure, and so does the outcome of any distance measurement procedure relying on sound waves [5]. Whereas the effects of humidity and pressure are negligible from our point of view, a 1°C change in air temperature near the 20°C range causes a substantial, 0.176% change in speed. The consequence of this for our BATSY system is that a sensor will measure an erroneous distance d instead of the real distance d/q, where q is some quotient depending on temperature.

Assuming we have distances measured by four sensors, the value of q can in theory be obtained by solving the following system of equations:

$$(x_1-x)^2+(y_1-y)^2+(z_1-z)^2=q^2d_1^2$$
$$(x_2-x)^2+(y_2-y)^2+(z_2-z)^2=q^2d_2^2$$
$$(x_3-x)^2+(y_3-y)^2+(z_3-z)^2=q^2d_3^2$$
$$(x_4-x)^2+(y_4-y)^2+(z_4-z)^2=q^2d_4^2$$

However, we have found that imprecision in the sensors' coordinates and other inaccuracies prevent this idea from being put into practice. So if our goal is to set up a system operating in room temperature, and unless we have temperature data from other sources (e.g. from an internal thermometer), it is better not to use this kind of temperature correction method altogether.

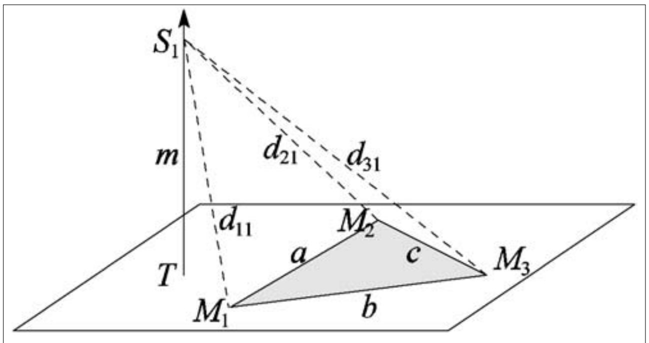
5. Determining the sensor's coordinates in an automated way

In order for the positioning system to yield an accurate result, the sensors' coordinates have to be measured with high precision. In a real-life deployment of the system, determining the sensors' exact position is the hardest and most time-consuming task. Using a traditional tape-measure or a laser rangefinder takes hours, and it leads to errors in the centimetres range. Is it possible to use the system itself for locating the sensors?

Obviously, it is impossible to have all the coordinates determined by the system, since the origin of the coordinate-system and the direction of the axes need to be chosen in some arbitrary way. (For convenience, we have chosen the vertical direction as the third axis of the coordinate system.) Thus our goal is to come to a self-configuring algorithm involving as little technical difficulty as possible and capable of determining the sensors' coordinates in some chosen Cartesian coordinate system. The algorithm we are about to discuss requires the following input data (these must be determined manually):

1. Coordinates of an arbitrary sensor S_1.
2. One of the non-vertical coordinates of some other sensor S_2.
3. The side lengths and the orientation of a triangle arbitrarily drawn on some horizontal plane.

Figure 5. The tetrahedron $M_1M_2M_3S_1$



5.1 Description of the algorithm

Denote the mounted sensors by $S_1,S_2,...S_n$, and their coordinates by (x_1,y_1,z_1), (x_2,y_2,z_2), ... (x_n,y_n,z_n), respectively. Let M_1,M_2,M_3 be the vertices of our horizontal triangle, and let a,b,c denote the distances between them *(Figure 5)*. According to our initial assumption, x_1,y_1,z_1,x_2 (or y_2), a,b,c are known, and our goal is to determine x_i,y_i,z_i (i=1,...n). Without loss of generality, one can impose $x_1=0$, $y_1=0$, $z_1=0$.

Step 1:
Measurement

Place the mobile unit at location M_1, and let the system measure its distance from the sensors. (Let $d_{11},d_{12},...d_{1n}$ denote these n distances.) Repeat the process with M_2 and M_3 so to obtain all the distances d_{ij} (i=1,2,3; j=1,...n).

Step 2:
Calculating the distance between S_1 and the plane $M_1M_2M_3$

First, calculate the volume of a tetrahedron with M_1, M_2,M_3,S_1 as its vertices. This can be done in two different ways. On one hand, according to Tartaglia's formula [6], we have

$$V^2=\frac{1}{288}\det\begin{bmatrix} 0 & d_{11}^2 & d_{21}^2 & d_{32}^2 & 1 \\ d_{11}^2 & 0 & a^2 & b^2 & 1 \\ d_{21}^2 & a^2 & 0 & c^2 & 1 \\ d_{31}^2 & b^2 & c^2 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$

On the other hand (using the notation $s=(a+b+c)/2$), the area of the base triangle is, by Heron's formula [6], as follows

$$T=\sqrt{s(s-a)(s-b)(s-c)}$$

Then, we have $V=Tm/3$, where m is the height of our tetrahedron as measured from the base triangle $M_1M_2M_3$. Making use of the equivalence of these two expressions for V, one can easily calculate m, which is precisely the distance between S_1 and the plane $M_1M_2M_3$.

Step 3:
Calculating the relative positions of M_1,M_2,M_3

Denote by T the foot of the tetrahedron's altitude line connecting S_1 to the base triangle $M_1M_2M_3$ (i.e. T is the orthogonal projection of S_1 to the plane $M_1M_2M_3$). Let r_1,r_2,r_3 be the distances of M_1,M_2,M_3 from T *(Figure 6 – on the next page)*. From the Pythagorean theorem, we have

$$r_1=\sqrt{d_{11}^2-m^2},\quad r_2=\sqrt{d_{21}^2-m^2},\quad r_3=\sqrt{d_{31}^2-m^2}.$$

Now fix a two-dimensional Cartesian coordinate system on the plane $M_1M_2M_3$, with T as its origin and one of its axes going through M_1. As r_1 denotes the distance $\overline{TM_1}$, point M_1 has coordinates $(0,r_1)$ in the afore mentioned system.

Similarly, r_2 denoting the distance $\overline{TM_2}$ and a denoting the distance $\overline{M_2M_1}$, coordinates (x,y) of point M_2 are obtained as the solution to the following system of equations

$$x^2+y^2=r_2^2$$
$$x^2+(y-r_1)^2=a^2$$

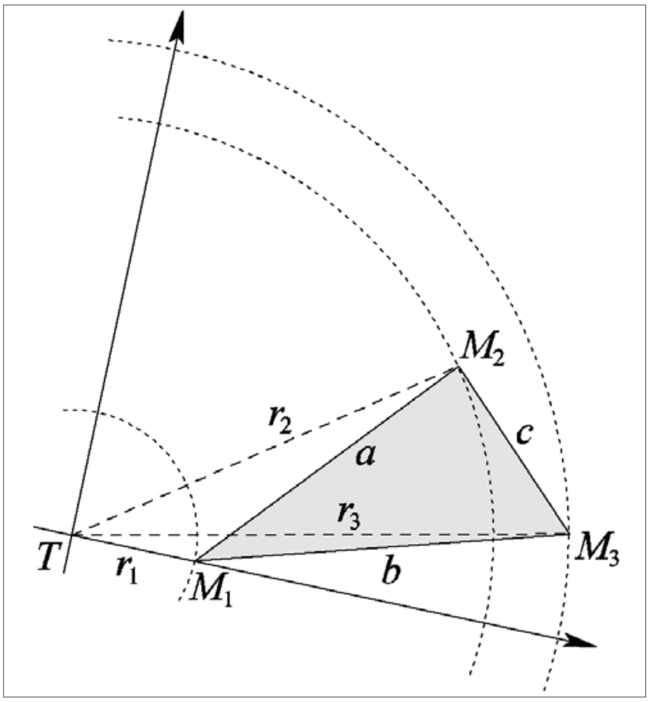


Figure 6. Orthogonal projection to the plane $M_1M_2M_3$

The above equation system has two solutions, among which the correct one is to be chosen in accordance with the orientation of triangle $M_1M_2M_3$.

The same method can be used for determining the coordinates of M_2. However, in this case the correct one out of the two solutions is to be selected imposing the further restriction that the distance $\overline{M_2M_3}$ must be equal with c.

The origin of this particular coordinate system is obtained through an orthogonal projection of the original system to plane $M_1M_2M_3$, yet the directions of their axes are different. Thus, we have so far determined the positions of M_1, M_2, M_3 relative to the new coordinate system, which we further need to rotate around the axis S_1T in order to get their coordinates in the original one.

Step 4:
Determining the angle of rotation around the axis S_1T

Let us return to the three-dimensional space. The two-dimensional relative coordinates of M_1, M_2, M_3 (as determined in Step 3) need to be complemented with a third one, which can be expressed as the negative distance between plane $M_1M_2M_3$ and point S_1, that is, $(-m)$.

Starting from these coordinates, and making use of the distances d_{12}, d_{22}, d_{32}, the coordinates of S_2 in the rotated system can be calculated through the trilateration procedure discussed in Section 2. Its two candidate solutions being symmetrical with respect to plane the $M_1M_2M_3$, we need to choose the one which lies on the same side of the plane as where the sensor is actually located.

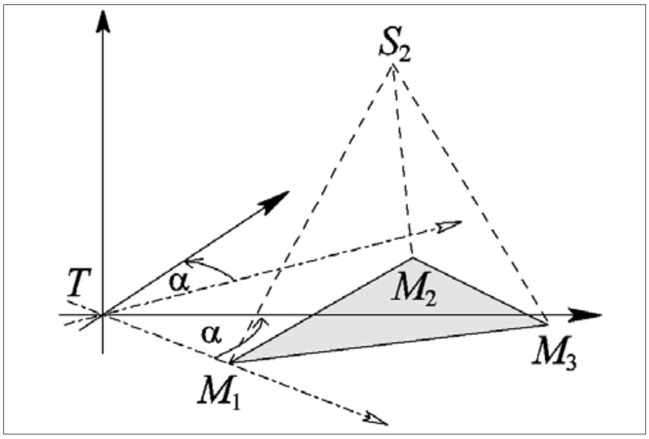
Denote by (x'_2, y'_2, z'_2) the coordinates of S_2 in the rotated system. Its coordinates in the original system are (x_2, y_2, z_2) where only x_2 is known for the present. Following from the properties of the rotated coordinate system, we have $z'_2 = z_2$.

If we transliterate the triplet (x'_2, y'_2, z'_2) to a different coordinate system derived from the original one through a rotation by α around the vertical axis *(Fig. 7)*, the new coordinates will be $(x'_2\cos\alpha - y'_2\sin\alpha,\ x'_2\sin\alpha + y'_2\cos\alpha,\ z'_2)$. From the equality of the first coordinates in the two systems, one easily comes to the trigonometrical equation

$$x_2 = x'_2\cos\alpha - y'_2\sin\alpha$$

This equation has two solutions for α, thus yielding two candidates for S_2, which lie symmetrically with respect to the plane parallel to axes (z,x) and containing S_1. Again, we have to select the correct α, the one corresponding to the particular candidate for S_2 which is located closer to its real position.

Figure 7.
Rotation from the temporary coordinate system to the original one



Step 5:
Calculating the coordinates of M_1, M_2, M_3 in the original coordinate system

In order to come to the absolute coordinates of M_1, M_2, M_3, their relative coordinates (as calculated in Step 3) need to be multiplied by the matrix

$$\begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

where α is the one determined in Step 4.

Step 6:
Obtaining the coordinates of sensors $S_2, \ldots S_n$

From Step 5 we know the positions of M_1, M_2, M_3, along with their distances from S_i: d_{1i}, d_{2i}, d_{3i}. Thus the coordinates of $S_2, \ldots S_n$ can be determined using the trilateration algorithm discussed in Section 3.

5.2 Practical considerations

Since the system's overall precision depends highly on the accuracy of the sensors' coordinates, it is essential to reduce any distortions relative to the process as much as possible.

1. The impact of temperature on the speed of sound can be offset using a reference measure taken at the beginning of the process (calibration). Place the mobile

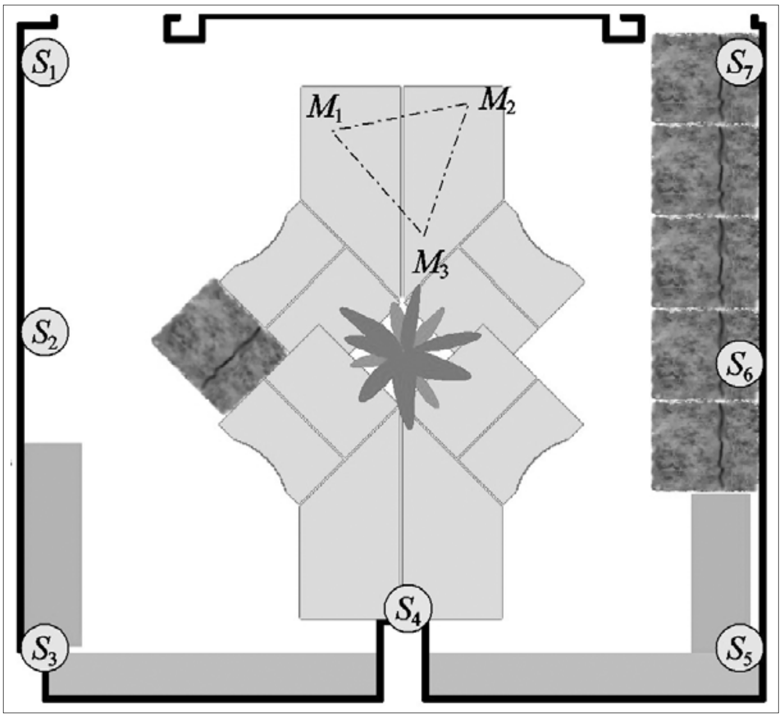


Figure 8. BATSY test configuration

node to a known benchmark distance from any particular sensor, and let the system measure its distance. Then, through multiplying all measured distances by the quotient of the afore measured and the real (benchmark) distances, the distorting effect of temperature is eliminated.

2. The inaccuracy related to the sensors' directional angle can be reduced through directing the sensors towards the triangle $M_1M_2M_3$.

3. The bias related to the mobile node's directional angle can be reduced using the method discussed in Section 4, assuming the ultrasound emitters are directed vertically while localising M_1, M_2, M_3.

4. The imprecision relative to the wavelength of the 40 kHz ultrasound (as discussed in Section 4.2) can practically be eliminated through taking the average of several measured distances.

5. There might be sensors whose positions cannot be determined by algorithm 4.1 (e.g. if points M_1, M_2, M_3 happen to be out of sight of a particular sensor). In this case, the already localised sensors can be used for determining the positions of some additional points M_4, M_5, M_6 and then for localising further sensors through Step 6 of the algorithm.

5.3 Test results

The algorithm described in section 5.1 and further amended in line with the considerations discussed in Section 5.2 was tested in a room 5x6 metres in area and 2.8 metres in height. Points M_1, M_2, M_3 were fixed on the surface of a 72 cm high desk, each of them situated 1 meter away from any other. We had put all three coordinates of sensor S_1 along with coordinate y of sensor S_7 into the system, and tried to determine the positions of sensors S_2,...S_7 *(Figure 8)*.

Sensor S_3 had no sight of view to points M_i so the algorithm couldn't estimate its position. A plant was obstructing the path between S_4 and points M_i, so we expected incorrect values for its coordinates. The results are shown in *Table 1*. One can see the accuracy of the method is good enough to determine the sensor's initial positions to use in the BATSY localisation system – if there are no obstacles along the ultrasound's path.

6. Conclusions

In this paper we presented the BATSY ultrasonic localisation system, discussed the positioning algorithm, observed the localisation errors resulting from possible distortions in distance measurement, and proposed ways of reducing them. We have provided an algorithm for configuring the system semi-automatically, and tested the results one can expect when operating the system in real-life conditions. The measurements confirmed that our configuring algorithm can be used to determine the sensors' position in ideal circumstances, however, when obstacles blocked the ultrasound's path, higher errors appeared.

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Table 1. Test results

Sensor	Real coordinates *(cm)*			Calculated coordinates *(cm)*			Error *(cm)*
S_1	32.5	24.5	280.8	32.5	24.5	280.8	0
S_2	31.2	241.9	278.0	30.3	240.2	278.6	1.98
S_3	37.2	501.1	280.6	n.a.	n.a.	n.a.	n.a.
S_4	305.9	474.1	281.8	301.3	467.0	293.5	14.44
S_5	583.3	500.2	280.5	583.8	498.2	282.6	2.97
S_6	584.0	278.7	278.6	582.1	278.3	280.4	2.59
S_7	582.7	27.2	280.3	578.8	27.2	285.6	6.64

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